MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 3, 2016/2017

TMA 1201 – DISCRETE STRUCTURES AND PROBABILITY

(TC01 / TT01)

26 MAY 2017
9.00 a.m. – 11.00 a.m.
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This Question paper consists of **6 pages** only excluding the cover page with **4 Questions** and an **Appendix**.

2. Attempt **ALL FOUR** questions. The total marks for this paper is 50. Distribution of all the marks for each question is given.

3. Please write your answers in the Answer Booklet provided, and **START EACH QUESTION ON A NEW PAGE**.

4. Show all steps clearly.

Question 1 (Total Marks: 10)

SET, RECURSION, ALGORITHM

a. Given the following sets:

$X = \{x \mid x \text{ is positive even integer less than } 10\}$
$Y = \{x \mid x \text{ is positive odd integer less than } 10\}$
$Z = \{x \mid x \text{ is a prime number less than } 10\}$

i. List the **elements** of set operation $(X \cup Y) \cap Z$. ($2\frac{1}{2}$ marks)

ii. Is Z a **subset** of $(X \cup Y) \cap Z$? Explain your answer. ($1\frac{1}{2}$ marks)

b. Complete the **analysis table** below by writing the number of times each line is being executed: (4 marks)

Line No.	Code	**Execution Time**
1	`int t0 = 0; int t1 = 1; int nextTerm = 0; int n;`	
2	`cout << "Number of terms is: " << n << endl;`	
3	`cout << "Base Values:" << t0 << ", " << t1 << ", " << endl;`	
4	`for (int i = 2; i <= n ; i++)`	
5	`{ nextTerm = t0 + t1;`	
6	`t0 = t1;`	
7	`t1 = nextTerm;`	
8	`cout << "Recursive Term: " << nextTerm << " " << endl; }`	

c. Write down the **output** of the code in (b) if `n = 3`. (2 marks)

Continued...

Question 2 **(Total Marks: 10)**

LOGIC, FORMAL REASONING

a. Given the following **premises**:

Premise 1: "If the perimeter of a square ABCD is 40cm, then each side of square ABCD is 10cm."

Premise 2: "If each side of square ABCD is 10cm, then the area of square ABCD is $100cm^2$."

Premise 3: "The area of square ABCD is not $100cm^2$."

i. Define 3 **propositions as** ***p, q, r*** to describe the premises above. ($1\frac{1}{2}$ marks)

ii. **Translate** each premises above into compound propositions with suitable logical connectives. ($1\frac{1}{2}$ marks)

iii. Prove by **formal reasoning** that the conclusion is:

"The perimeter of square ABCD is not 40cm." (4 marks)

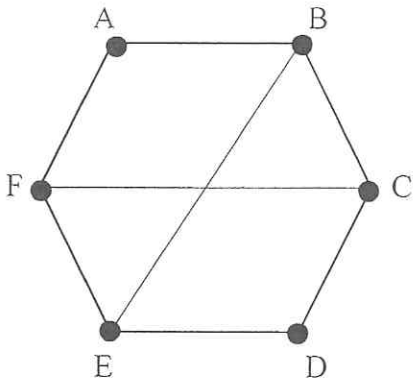
b. State the **converse, inverse**, and **contrapositive** of Premise 1 in (a). (3 marks)

Continued...

Question 3 **(Total Marks: 15)**

GRAPH THEORY, TREE, FINITE-STATE AUTOMATA

a. Given the following Graph W:

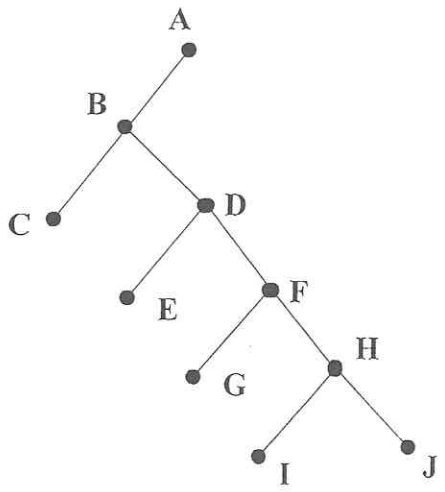


Graph W

i. Create the **adjacency matrix** for Graph W. (3 marks)

ii. Does Graph W contains an **Euler Circuit**? Explain your answer. (2 marks)

b. Given the following **Binary Tree** with vertex A as the root:



i. Write down the **post-order** traversal for the Binary Tree. (2 marks)

ii. State the **height** of the Binary Tree. (1 mark)

Continued...

c. The following is a table that describes a finite-state automaton with **an initial state at S_0 and an accepting state at S_1:**

State \ Input	Next State	
	0	1
S_0	S_1	S_2
S_1	S_0	S_2
S_2	S_0	S_3
S_3	S_1	S_3

i. Draw the **state-transition diagram** of the finite-state automaton. (4 marks)

ii. Complete the table below to decide if the automaton accepts or rejects the string **1001**. (2 marks)

Inputs	1	0	0	1
Next-States				

iii. Give an example of **a 4-digit string** that will be accepted by the automaton. (1 mark)

Continued...

Question 4 **(Total Marks: 15)**

COMBINATORICS, DISCRETE PROBABILITY AND DISTRIBUTION

a. In an annual sport event at a university, there are indoor games and outdoor games that students can take part. The following facts were found from previous event:

The probability for a student to join an outdoor game is 0.75.
The probability for a student to join an indoor game is 0.35.
The probability for a student to join both indoor and outdoor games is 0.25.

i. Draw a **Venn Diagram** to illustrate these probabilities. (1 mark)

ii. Find the probability that a student joins **at least a game** in the sport event this year. (2 marks)

iii. Find the probability that a student joins **neither** an indoor nor an outdoor game in the sport event this year. (2 marks)

b. The following table describes the probability density function (pdf) of a random variable X at a store. Given that X describes the number of eggs that is found to be cracked in a small container of maximum 6 eggs at the store:

No. of cracked eggs x	0	1	2	3	4	5	6
Probability of x, $P(x)$	0.5	m	m	0.04	0.03	0.02	0.01

i. Find the value of m. (2 marks)

ii. Calculate the expected value, **E[X]** for the pdf. (2 marks)

iii. Calculate the variance, **V[X]** for the pdf. (3 marks)

iv. Find the probability that **not more than one egg** is found to be cracked. (1 mark)

v. Given that **finding no cracked eggs at all at the store is a success**, find the probability that in a sample of 12 containers, **at most** 11 containers were found not to have any cracked eggs at all. (2 marks)

End of Questions.

Appendix

Name	Tautological Arguments	Premises	Conclusion
Addition	$p \Rightarrow p \vee q$	1. p	$p \vee q$
Simplification	$(p \wedge q) \Rightarrow p$	1. p 2. q	p
Modus Ponens	$[p \wedge (p \rightarrow q)] \Rightarrow q$	1. p 2. $p \rightarrow q$	q
Modus Tollens	$[\neg q \wedge (p \rightarrow q)] \Rightarrow \neg p$	1. $\neg q$ 2. $p \rightarrow q$	$\neg p$
Hypothetical Syllogism	$[(p \rightarrow q) \wedge (q \rightarrow r)]$ $\Rightarrow (p \rightarrow r)$	1. $p \rightarrow q$ 2. $q \rightarrow r$	$p \rightarrow r$
Disjunctive Syllogism	$[(p \vee q) \wedge \neg p] \Rightarrow q$	1. $p \vee q$ 2. $\neg p$	q
Resolution	$[(p \vee q) \wedge (\neg p \vee r)]$ $\Rightarrow (q \vee r)$	1. $p \vee q$ 2. $\neg p \vee r$	$q \vee r$
Conjunction	$p \wedge q \Rightarrow (p \wedge q)$	1. p 2. q	$p \wedge q$